QUALIFYING EXAMINATIONS STUDY GUIDE

September 2007

1. ABSTRACT ALGEBRA

   (a) Integers

      i. Euclidean Algorithm
      ii. Greatest common divisor and least common multiple
      iii. Fundamental Theorem of arithmetic
      iv. Euler phi-function

   (b) Groups

      i. Definition of groups and subgroups, Lagrange's Theorem
      ii. Cyclic groups, dihedral groups
      iii. Symmetric groups, alternating groups, factorization into a product of disjoint cycles
      iv. Direct products
      v. Fundamental Theorem on Finite Abelian Groups
      vi. Matrix groups - General linear group and other groups involving matrices;
      vii. Homomorphisms, monomorphisms, epimorphisms, and isomorphisms
      viii. Normal subgroups quotient groups
      ix. Cayley's Theorem (every group is isomorphic to a permutation group)
      x. Definition of simple groups
      xi. Knowledge of all groups of small order - say up to 10.

   (c) Rings and Fields

      i. Definition of rings and subrings
      ii. Homomorphisms
      iii. Ideals and quotient rings
      iv. Definition of integral domain and field
      v. Fields $\mathbb{Z}/p\mathbb{Z}$
      vi. Field of fractions of an integral domain
      vii. Principal ideal domains
      viii. Polynomial rings $\mathbb{R}[x]$, irreducible polynomials
      ix. Euclidean domains and unique factorization domains
      x. Calculations in $(\mathbb{Z}/p\mathbb{Z})[x]$ and reduction modulo n

2. LINEAR ALGEBRA

   (a) Linear Equations, Vector Spaces, Linear Transformations

      i. Systems of linear equations, matrices, elementary row operations, reduced row echelon form of a matrix

ii. Matrix algebra.
iii. Vector spaces, subspaces
iv. Bases and dimension
v. Coordinate matrix relative to a basis
vi. Linear transformations/linear mappings
vii. Representation of transformations by matrices
viii. Bilinear forms, symmetric bilinear forms (scalar product), orthogonality, positive definite case, orthonormal bases
ix. Sylvester therorem
x. Bilinear maps, general orthogonal bases, quadratic forms
xi. Inner product spaces, linear functionals, normal operators

(b) Determinants and Canonical Forms

i. Definition and properties of determinants
ii. Eigenvalues, characteristic and minimal polynomials, Cayley-Hamilton Theorem
iii. Invariant subspaces, simultaneous triangularization and diagonalization
iv. Cyclic subspaces and decompositions
v. Rational and Jordan forms, computation of invariant factors

3. FUNCTIONS OF ONE VARIABLE

(a) Continuity
(b) Sequences and series of numbers
(c) Sequences and series of functions
(d) Uniform convergence
(e) Differentiation
(f) The mean value theorem
(g) Taylor's series
(h) The Riemann integral
(i) The fundamental theorems of calculus
(j) Uniform convergence in relation to continuity, integration and differentiation
(k) The Stone-Weierstrass theorem (uniform approximation of continuous functions by polynomials on closed intervals)
(l) Equicontinuity and the Arzela-Ascoli theorem

4. FUNCTIONS OF SEVERAL VARIABLES

(a) Linear transformations on $\mathbb{R}$ and identification with matrices
(b) Differentiation of functions of several variables

(c) Partial derivatives and their relationship to differentiation

(d) The mean value theorem

(e) Taylor's series

(f) The inverse function theorem

(g) The implicit function theorem

(h) Determinants and higher order derivatives

(i) Change of variables formula and Jacobians

(j) Differentiation under the integral sign

5. COMPLEX ANALYSIS

(a) Complex differentiation and the Cauchy-Riemann equations

(b) Harmonic functions

(c) The maximum and minimum modulus principles

(d) The open mapping theorem

(e) Integration over paths

(f) Cauchy's theorem and power series representation

(g) Taylor and Laurent series

(h) Calculus of residues

(i) The Schwarz lemma

(j) Conformal mappings

(k) Singularities

(l) Rouché's theorem

(m) The Casorati-Weierstrass theorem

6. RECOMMENDED READING

First and foremost we recommend that you contact the teachers of the current or previous Math 8210 (Basic Algebra) and Math 8220 (Basic Analysis) classes.

(a) Abstract and Linear Algebra

Math 4720 (Introduction to Abstract Algebra I):

i. Herstein: Abstract Algebra 3rd. ed.
Sections: 1.1-7, 2.1-7, 2.9-10, 3.1-3, 4.1-7, 5.1, 5.3-4

ii. Fraleigh: A First Course in Abstract Algebra 5th. ed.
Sections: 1.1-5, 2.1-3, 3.1-4 5.1-6, 6.1-2, 7.1-3, 8.1-3, 8.5

Math 4920 (Introduction to Abstract Linear Algebra):

i. Lang: Linear Algebra 3rd. ed. (+ Solutions Manual by Shakarchi)
Chapters: I-XI

ii. Hoffman and Kunze: Linear Algebra 2nd. ed.

(b) Real and Complex Analysis

Math 4940 (Introduction to Complex Variables):

i. Brown and Churchill: Complex Variables and Applications 6th. ed.
Chapters: 1-9

ii. Marsden and Hoffman: Basic Complex Analysis 2nd. ed.
Chapters: 1-6

Math 4700 (Advanced Calculus I):

i. Wade: Introduction to Analysis
Chapters: 1-4, excluding section 3.5

ii. Bartle and Sherbert: Introduction to Real Analysis, 2nd. ed.

iii. Rudin: Principles of Mathematical Analysis, 3rd. ed.
Chapters: 1-5, 7

Math 4900 (Advanced Calculus II):

i. Rudin: Principles of Mathematical Analysis, 3rd. ed.
Chapters: 9, 10 (Sections: 10.1,10.2,10.4)

ii. Wade: Introduction to Analysis
Chapters: 6-7