Qualifying Examination II **August 19, 1998**

If you have any difficulty with the wording of the following problems please contact the supervisor immediately. All persons responsible for these problems, in principle, will be accessible during the entire duration of the exam.

Advanced Calculus I

1. (a) (4 points) Let $g : [a, b] \to \mathbb{R}$ be a continuous function. Show that if g has infinitely many zeros in $[a, b]$, then there exist $x_0 \in [a, b]$ and a sequence $\{x_n\}$ in $[a, b]$ such that $x_0 = \lim_{n\to\infty} x_n$, $g(x_0) = 0$, and $g(x_n) = 0$ for all $n \in \mathbb{N}$.

(b) (6 points) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Assume f and f' have **no common zeroes**. Prove that f has only finitely many zeros in $[0, 1]$.

2. (a) (5 points) Let $f : [0, \infty) \to \mathbb{R}$ be the function $f(x) = \sqrt{x}$. Prove that f is uniformly continuous on $[0, \infty)$.

(b) (5 points) Let $k > 0$. Define the function $g(x) = \dfrac{(x - x^k)}{\log(x)}$ for $0 < x < 1$, $g(0) = 0$ and $g(1) = 1 - k$. Show that g is uniformly continuous on $[0, 1]$. (Hint: Show that g is continuous on $[0, 1]$.)