

QUALIFYING EXAMINATIONS STUDY GUIDE

April 2002

1 Algebra: Groups, Rings, Fields

1.1 Integers

1. Euclidean Algorithm
2. Greatest common divisor and least common multiple
3. Fundamental Theorem of arithmetic
4. Euler phi-function

1.2 Groups

1. Definition of groups and subgroups, Lagrange's Thm
2. Cyclic groups, dihedral groups
3. Symmetric groups, alternating groups, factorization into a product of disjoint cycles
4. Direct products
5. Matrix groups - General linear group and other groups involving matrices;
6. Homomorphisms, monomorphisms, epimorphisms, and isomorphisms
7. Normal subgroups, quotient groups
8. Cayley's Thm (every group is isomorphic to a permutation group)
9. Definition of simple groups
10. Knowledge of all groups of small order - say up to 10.

1.3 Rings and Fields

1. Definition of rings and subrings
2. Homomorphisms
3. Ideals and quotient rings
4. Definition of integral domain and field
5. Fields $\mathbb{Z}/p\mathbb{Z}$
6. Field of fractions of an integral domain
7. Principal ideal domains
8. Polynomial rings $\mathbb{R}[X]$, irreducible polynomials
9. Euclidean domains and unique factorization domains
10. Calculations in $(\mathbb{Z}/p\mathbb{Z})[X]$ and reduction modulo n

2 Algebra: Linear Algebra

2.1 Linear Equations, Vector Spaces, Linear Transformations

1. Systems of linear equations; matrices and elementary row operations; Reduced row echelon form of a matrix
2. Matrix algebra.
3. Vector spaces; subspaces
4. Bases and dimension
5. Coordinate matrix relative to a basis
6. linear transformations/linear mappings
7. Representation of transformations by matrices
8. Bilinear forms, symmetric bilinear forms (scalar product), Orthogonality, positive definite case, orthonormal bases.
9. Sylvester theorem
10. Bilinear maps, general orthogonal bases, quadratic forms
11. Innerproduct spaces, linear functionals, normal operators.

2.2 Determinants and Canonical Forms

1. Definition and properties of determinants
2. Eigenvalues; characteristic and minimal polynomials Cayley-Hamilton Thm
3. Invariant subspaces; simultaneous triangularization and diagonalization
4. Cyclic subspaces and decompositions
5. Rational and Jordan forms; computation of invariant factors

3 FUNCTIONS OF ONE VARIABLE

1. Continuity
2. Sequences and series of numbers
3. Sequences and series of functions
4. Uniform convergence
5. Differentiation
6. The mean value theorem
7. Taylor's series
8. The Riemann integral
9. The fundamental theorems of calculus
10. Uniform convergence in relation to continuity, integration, and differentiation
11. The Stone-Weierstrass theorem (uniform approximation of continuous functions by polynomials on closed intervals)
12. Equicontinuity and the Arzela-Ascoli theorem

4 FUNCTIONS OF SEVERAL VARIABLES

1. Linear transformations on \mathbb{R}^n and identification with matrices
2. Differentiation of functions of several variables
3. Partial derivatives, and their relationship to differentiation
4. The mean value theorem
5. Taylor's series

6. The inverse function theorem
7. The implicit function theorem
8. Determinants and higher order derivatives
9. Change of variables formula and Jacobians
10. Differentiation under the integral sign

5 COMPLEX ANALYSIS

1. Complex differentiation and the Cauchy-Riemann equations
2. Harmonic functions
3. The maximum and minimum modulus principles
4. The open mapping theorem
5. Integration over paths
6. Cauchy's theorem and power series representation
7. Taylor and Laurent series
8. Calculus of residues
9. The Schwarz lemma
10. Conformal mappings
11. Singularities
12. Rouché's theorem
13. The Casorati-Weierstrass theorem

6 Recommended Reading

First and foremost, we recommend that you contact the teachers of the current or previous Math 401 and 402 classes.

6.1 Algebra and Linear Algebra

Math340:

Herstein: Abstract Algebra, 3rd. ed.
Sections: 1.1-7, 2.1-7, 2.9, 3.1-3, 4.1-7, 5.1, 5.3-4
Fraleigh: A First Course in Abstract Algebra, 5th. ed.
Sections: 1.1-4, 2.1-4, 3.1-4, 5.1-6, 6.1-2, 7.1-3, 8.1-3, 8.5

Math341:

Lang: Linear Algebra, 3rd. ed. (+ Solutions Manual by Shakarchi)
Chapters: I-XI
Hoffman and Kunze: Linear Algebra, 2nd. ed.
Chapters: 1-9 and Appendices: A.4-A.5

6.2 Real and Complex Analysis

Math305:

Brown and Churchill: Complex Variables and Applications, 6th. ed.
Chapters: 1-9
Marsden and Hoffman: Basic Complex Analysis, 2nd. ed.
Chapters: 1-6

Math310:

Wade: Introduction to Analysis
Chapters: 1-4, excluding section 3.5
Bartle and Sherbert: Introduction to Real Analysis, 2nd. ed.
Rudin: Principles of Mathematical Analysis, 3rd. ed.
Chapters: 1-5, 7

Math311:

Rudin: Principles of Mathematical Analysis, 3rd. ed.
Chapters: 9, 10 (Sections: 10.1,10.2,10.4)
Wade: Introduction to Analysis
Chapters: 6-7