

Due Date: Friday 11 April 2008

1. Consider the initial value problem

$$\dot{x} = x(1 - x), \quad x(0) = 1/100.$$

- (a) Find the exact solution of the initial value problem.
 - (b) Implement Euler's method to solve this differential equation and demonstrate by numerical experiments that Euler's method is order one.
 - (c) Implement the Improved Euler's method to solve this differential equation and demonstrate by numerical experiments that it is order two.
 - (d) Use a computer to draw a graph of the exact solution and the approximate solution (obtained using Euler's method) of the differential equation on the interval $[0, 10]$.
 - (g) Is the improved Euler method better? Here "better" means "able to obtain the same accuracy with fewer steps." (Hint: Consider step sizes on the order of $1/10$.)
2. The differential equation in this homework can be interpreted to represent the limited growth of a population x , say fish in a lake. Maybe x is measured in thousands of pounds of fish. Suppose we start fishing! A simple model is

$$\dot{x} = x(1 - x) - a,$$

where a is a measure of the amount of fishing (i.e. the amount of fish that are continuously removed). Let's assume that we start with $x(0) = 5$.

- (a) Do some computer experiments using the Improved Euler method to determine the effect of fishing by considering a in the range $[0, 1/2]$. Integrate over 5 units of time. You can try other values if you wish. Write out in words what you learn. There is a critical value of a where something dramatic happens? What is this value.

(b) Explain, using pencil and paper, why a critical value exists.

3. For the initial value problem

$$\dot{x} = x(1 - x) + 1 - \frac{1}{10} \sin t, \quad x(0) = 2,$$

find the value of $x(6)$ correct to three decimal places. Explain why you believe your answer is correct!

You are allowed to work in groups. But, the work you hand in should be written in your own words. Important: You must explain what you see in the numerical experiments using complete sentences. You are free to use your favorite computer system. If you like Mathematica, you might find useful the Mathematica Notebook on our course web-site available at the link “Mathematica File for Euler and Improved Euler.” This file has all the ingredients that are needed to do the homework assignment, but the file is **not set up to integrate the model in this assignment**. You will have to make appropriate modifications. On the other hand, if you read the Mathematica file and look at the output it creates, you might learn something.