Nonlinear Dynamics Topics

The concept of the course is to supplement ODE and PDE graduate courses with standard topics from dynamical systems theory that are only encountered in second semester and topics courses. The best way to learn this material is to make presentations and to do exercises. We will do both. In particular, all students taking the course for credit will be expected to make several presentations during the semester.

Some of the topics that will be discussed are listed below:

- Warmup Exercises (Do we already know basic ODE?)
- Euler-Lagrange (This will be a topic for a seminar.)
- Galërkin Approximation (Carmen)
- Traveling Waves (Mikael)
- First Order PDE (Geetha)
- Hyperbolic Theory
  - Contraction Mapping, Uniform Contraction, Fiber Contraction (Jamie)
  - Implicit Function Theorem and Robbin's proof of Existence Theorem (Mita)
  - Invariant Manifolds (Mike)
  - Hartman-Grobman Theorem (Oksana)
- Continuation Theory and Regular Perturbation Theory (In particular, Poincaré's method of the small parameter)
  - Autonomous Perturbations (Valerian)
  - Lyapunov Schmidt and Nonautonomous Perturbations (Alex)
- Bifurcation Theory (Kenny)
- Singular Perturbation Theory
- Normal Forms
- Chaotic Dynamics (Smale Horseshoe, Conley Moser Conditions, ...)

- Hamiltonian Dynamics (Action Angle Variables, Canonical Transforms, Arnold Diffusion, . . .)
- Ergodic Theory

Some standard reference books:

- Stephen Wiggins, Introduction to Applied Nonlinear Dynamical Systems. A good book on all aspects of nonlinear dynamics.
- John Guckenheimer and Philip Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcation Theory (A classic¿)
- Jürgen Moser, Stable and Random Motions (Hamiltonian Mechanics at its best.)
- Carl Siegel and Jürgen Moser, Lectures in Celestial Mechanics (Unusual style but classic treatment of celestial mechanics.)
- Ken Meyer and Dick Hall, Introduction to Hamiltonian Dynamical Systems and the N-body Problem
- Vladimir A'rnold, All his books, especially, Ordinary Differential Equations, Mathematical Methods of Classical Mechanics, Geometrical Methods in Ordinary Differential Equations (Arnold is a great teacher; but, he leaves out details in his books. They are a great place to learn the concepts but not the details of proofs.)
- Robert Devaney, An Introduction to Chaotic Dynamical Systems (One dimensional dynamical systems.)
- D. Arrowsmith and C. Place, An Introduction to Dynamical Systems
- Michael Brin and Garrett Stuck, Introduction to Dynamical Systems (This is a very nice (new) book for pure dynamical systems theory.)
- Larry Perko, Differential Equations and Dynamical Systems (Good graduate ODE book in competition Ordinary Differential Equations with Applications by Chicone).
- Ralph Abraham, Jerry Marsden and Tutor Ratiu, Manifolds, Tensor Analysis and Applications (An absolutely great reference book for global analysis. This is highly recommended.)

- Morris Hirsch and Steve Smale, Differential Equations, Dynamical Systems and Linear Algebra. A basic "undergraduate book" on dynamical systems.

- Coddington and Levinson, Theory of Ordinary Differential Equations (Classic ODE Book).

- Jack Hale, Several books, especially Ordinary Differential Equations (a difficult but classic book on ODE)

- Andronov, Leontovich, Gordon and Maier, Qualitative Theory... and Theory of Bifurcation of Dynamical Systems. Everything you every wanted to know about dynamical systems on the plane.

- A. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems. An encyclopedia of discrete dynamical systems theory.

- Jan Sanders and F. Verhulst, Averaging Methods in Nonlinear Dynamical Systems. (The basic book on averaging)

- Jim Murdock, Perturbations: Theory and Methods (Very well written undergraduate text on perturbation theory.)

- Clark Robinson, An Introduction to Dynamical Systems: Continuous and Discrete. (A good graduate book on dynamical systems.)