

Math 426 Homework 10

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1. Find the principal fundamental matrix solution at $t = 0$ for the Markus-Yamabe system: $\dot{x} = A(t)x$, where $A(t)$ is given on page 171. Write the principal fundamental matrix solution in real Floquet normal form. Find the characteristic multipliers of the system.
2. Suppose that $t \mapsto A(t)$ and $t \mapsto M(t)$ are $n \times n$ -matrix functions such that

$$\frac{d}{dt}A(t) = M(t)A(t) - A(t)M(t),$$

and let $\Phi(t)$ denote the principal fundamental matrix solution at $t = 0$ for $\dot{x} = M(t)x$. Prove that $A(t) = \Phi(t)A(0)\Phi^{-1}(t)$ and conclude that $A(t)$ is isospectral (the spectrum does not depend on t). Here (A, M) is called a Lax pair. Find M so that (A, M) is a Lax pair for the Markus-Yamabe system.

3. The proof of Theorem 2.47 (page 163) has a mathematical error. Find the error and correct the proof.