Math 426 Homework 3

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1. Exercise 1.34 p. 26 of the book. Consider a Newtonian particle of mass m moving under the influence of the potential U. If the position coordinate is denoted by

$$q = (q_1, \ldots, q_n),$$

then the equation of motion ($F = ma$) is given by

$$m\ddot{q} = -\text{grad}\, U(q).$$

If q_0 is a strict local minimum of the potential, show that the equilibrium $(\dot{q}, q) = (0, q_0)$ is Lyapunov stable. Hint: Consider the total energy of the particle.

2. Determine the stability of the rest points of the following systems. Formulate properties of the unspecified scalar function g so that the system has a rest point at the origin which is respectively stable, asymptotically stable, and unstable.

   1. $\dot{x} = 2xy - x^3$,
      $\dot{y} = -x^2 - y^5$.
   2. $\dot{x} = y + xy^2 - x^3 + 2xz^4$,
      $\dot{y} = -x - y^3 - 3x^2y + 3yz^4$,
      $\dot{z} = -\frac{5}{2}y^2z^3 - 2x^2z^3 - \frac{1}{2}z^7$.

3. Exercise 1.13 p. 13. Determine the flow of the first order system

$$\dot{x} = y^2 - x^2, \qquad \dot{y} = -2xy.$$

Show that (almost) every orbit lies on an circle. Note that the flow gives rational parameterizations for the circular orbits. Hint: Define $z := x + iy$.