

Math 426 Homework 7

C. Chicone

November 4, 2003

1. Prove that the system

$$\dot{x} = x - y - x^3, \quad \dot{y} = x + y - y^3$$

has a unique globally attracting limit cycle on the punctured plane; that is, the plane with the origin removed.

2. Can a system of the form

$$\dot{x} = y, \quad \dot{y} = f(x) - af'(x)y,$$

where f is a smooth function and a is a parameter, have a limit cycle? Hint: Consider a Liénard transformation.

3. Suppose that γ is a periodic orbit of a smooth flow defined on \mathbb{R}^2 . Prove that γ surrounds a rest point of the flow. This is a good example where proving the obvious can be difficult! Hint: One possible proof uses the Poincaré-Bendixson theorem and Zorn's lemma. If you are not familiar with Zorn's lemma, look it up in an analysis book, a book on topology, or on the web. Define a partial ordering on the set of all limit cycles as follows: For two limit cycles Γ_1 and Γ_2 , define $\Gamma_1 \leq \Gamma_2$ if Γ_2 is surrounded by Γ_1 . It is interesting to note that Zorn's lemma is equivalent to the Axiom of Choice and is therefore equivalent to one of the axioms of (Zermelo-Frankel) set theory.