Math 4560 Homework for Chapter 3

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Due Thursday 17 September

1. Problems to hand in: 3.5.6 (In part (e), a mechanical analog is sufficient. But, if you are an electrical engineer your pride may require you to come up with a circuit model!) 3.5.7, 3.7.3

2. Study the differential equation (6) on page 66 numerically. To set this up, write the system as a first-order system

$$\begin{aligned}\frac{d\phi}{d\tau} &= v,\\ \frac{dv}{d\tau} &= \frac{1}{\epsilon}(-v - \sin\phi + \gamma \sin\phi\cos\phi)\end{aligned}$$

   and vectorize your Runge-Kutta program. Or, if you like, you may use a "black box" integration routine such as `NDSolve` in Mathematica or one of the ODE solvers in Matlab. The solutions much be plotted as parametrized curves in the ϕv-plane (use `ParametricPlot` in Mathematica). (a) Make a plot that demonstrates the two time scales in the problem when ϵ is small. You should see that trajectories move very fast in certain regions and slow in others. You should sneak up on this; that is, find a moderately small value of ϵ that demonstrates the result. I am not so much interested in the graphs as I am in the words you use to describe what is to be observed. (b) Verify by numerical experiment the existence of a pitchfork bifurcation.
   Note: Systems with two time scales are called stiff systems in numerical analysis. Runge-Kutta integrators don't work well for such systems in general (that is, the step size usually has to be taken very small to

get accurate results). Special numerical methods (for example Gear's Method) are more efficient (that is, require fewer steps to achieve the desired accuracy). In our special case, however, Runge-Kutta works reasonably well. We are not too concerned about efficiency since we are only solving the problem once!