

Math 4560 Homework

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Due Tuesday October 27.

1. (a) Use a computer to verify the bifurcation diagram given in Figure 7.3.7 page 209 for the system

$$\dot{x} = -x + ay + x^2y, \quad \dot{y} = b - ay - x^2y.$$

Show, by drawing computer generated phase portraits corresponding to some set of (a, b) points, that the bifurcation diagram proved in class correctly predicts the existence of limit cycles. How does the amplitude of the limit cycle change as the parameters approach the bifurcation curve? Hand in some computer generated pictures and as usual be sure to **explain in words what I am supposed to see in your pictures**. (b) Show that Hopf bifurcation occurs as the bifurcation curve is crossed. You may choose and work with your favorite curve of parameter values that crosses the bifurcation curve. Use the computer to show that eigenvalues cross the imaginary axis in an appropriate manner for Hopf bifurcation and using the result in Problem 8.2.12, determine if the Hopf bifurcation is critical or subcritical for the bifurcation that occurs along your curve of parameter values.

2. Show that the system

$$\dot{x} = y - x^3, \quad \dot{y} = -x - y^3$$

has no limit cycle. Hint: Find a Lyapunov function of the form $L = ax^2 + by^2$ for suitable a and b .

3. Use the Poincaré-Bendixson theorem to prove that the system

$$\dot{x} = x - y - x^3, \quad \dot{y} = x + y - y^3$$

has a limit cycle.

4. Consider the ODE

$$\ddot{x} + \mu\dot{x}(x^2 + ax^2 - b) + x = 0$$

for $\mu \ll 1$, $a > 0$ and $b > 0$. Use the method of averaging (as presented in class) to predict that this system has a limit cycle in the phase plane. Determine the approximate radius of the limit cycle as a function of the system parameters. Confirm your prediction of the radius by a computer experiment for the case $a = 2$ and $b = 7$.