

# Math 4560 Homework for Chapter 9

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Due Monday November 5

1. Consider the system of ODEs given by

$$\dot{x} = -y - z, \quad \dot{y} = x + \frac{2}{10}y, \quad \dot{z} = \frac{2}{10} - cz + xz$$

where  $c$  is a parameter and  $0 < c < 6$ .

2. Analyze the rest points with pencil and paper.
3. Describe the qualitative changes in the system as  $c$  increases. Please write your description in carefully constructed English prose using computer graphics to illustrate the main points. Be sure to look at the parameter values  $c = 2.5, 3.4, 4, 5, 5.1$  as part of your numerical exploration. It might be wise to look at plots in 3d as well as plots in 2d (for example, plots in the  $xy$ -plane), and also some plots in 1d (for example,  $x$ ,  $y$ , or  $z$  versus  $t$ ). For “small” values of  $c$ , the attractor is a limit cycle. For “large” values of  $c$  the system has a chaotic attractor. What I want is the best evidence you can find to show that this scenario occurs. For example, a chaotic attractor will exhibit sensitivity to initial data (the butterfly effect). Provide numerical evidence that this effect does occur for at least one parameter value where the system has a chaotic attractor and it does not occur for a case where the system has a limit cycle. Of course, there is a transition (as  $c$  changes) from the existence of a periodic attractor to the existence of chaos. What occurs during this transition? Are the features different during the transition, or do we go directly from a single limit cycle to a chaotic attractor.

4. Draw a graph of the Lorenz map for the case  $c = 5.1$ .
5. Notes: When looking for attractors, it is a good idea to throw away the transient and only plot the attractor. You have to test how long to integrate to be sure you are on the attractor, throw this part away, and plot the result of integrating further in time.
6. Remember that it is useless to show a bunch of graphs unless you explain what the reader is supposed to see! Be sure to say in words what the reader is supposed to see in your figures.
7. You are allowed to ask me questions about assignments. Please let me know if you do not understand what I want you to do.