

Math 304 problems IV

1. Find the solution of the IVP

$$y'' + y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

by converting to a first order matrix system and then solving the matrix system.

2. Solve the IVP for the system of first-order ODEs

$$\begin{aligned}\dot{x} &= -2x + 3y + 2, \\ \dot{y} &= 3x - 2y - 1\end{aligned}$$

with $x(0) = 1$ and $y(0) = 2$.

3. Find the general solution of the system of first-order ODEs

$$\begin{aligned}\dot{x} &= 2x - 5y - \cos t, \\ \dot{y} &= x - 2y + \sin t.\end{aligned}$$

4. Find the general solution of the system of first-order ODEs

$$\begin{aligned}\dot{x} &= 3x + 2y + \sin 2t, \\ \dot{y} &= -2x + 3y.\end{aligned}$$

5. Convert the ODE $\ddot{x} + \dot{x} + x - x^3 = 0$ to a first order system. Find all the rest points and draw the local phase portraits near the rest points.
6. The system

$$\begin{aligned}\dot{x} &= 3x - xy - x^2, \\ \dot{y} &= -y + xy - y^2\end{aligned}$$

is a model for the interaction of a prey species and a predator species. The variable x is a measure the population of the prey in a limiting environment; y a measure of the population of the predator. The interaction term is xy . Note that the interaction increases the predator population (hence the plus sign), but the interaction (getting eaten) decreases the population of the prey. What eventually happens to the populations?