

1 Dynamical Systems Zoo

- Quadratic Family

$$x_{n+1} = \lambda x_n(1 - x_n)$$

Try $\lambda \in [1, 4]$.

- Hénon Family

$$\begin{aligned}x_{n+1} &= y_n - ax_n^2 + 1, \\y_{n+1} &= bx_n.\end{aligned}$$

Try $a = 1.4$ and $b = 0.3$.

- Hénon's Quadratic Family

$$\begin{aligned}x_{n+1} &= x_n \cos \alpha - y_n \sin \alpha + x_n^2 \sin \alpha, \\y_{n+1} &= x_n \sin \alpha - y_n \cos \alpha - x_n^2 \cos \alpha.\end{aligned}$$

Try $\cos \alpha = 0.22$.

- Pendulum With Torque And Parametric Forcing

$$\ddot{\theta} + \mu \dot{\theta} + \sin \theta = -\tau + 2 \cos 2t \sin \theta.$$

Try $\tau = 0.1$ and $\mu \in [0.03, 0.65]$.

- Duffing With Periodic Forcing

$$\ddot{x} + \delta \dot{x} - x + x^3 = \gamma \cos \omega t.$$

Try $\delta = 0.25$, $\gamma = 0.3$, and $\omega = 1.0$.

- Van der Pol with Periodic Forcing

$$\ddot{x} + \delta(x^2 - 1)\dot{x} + x = \gamma \cos \omega t.$$

For the pendulum mark $(\theta, \dot{\theta})$ every π units of time. For the Duffing and van der Pol, mark (x, \dot{x}) every 2π units of time. In other words, plot iterates of the stroboscopic Poincaré map.

References

- [1] J. Gleick, *Chaos: The Birth of a New Science*, Viking Press, New York, 1987 (Best popular introduction).
- [2] E. N. Lorenz, Deterministic nonperiodic flow, *J. Atmos. Sci.* 20 (1963) 130–141 (Famous paper).
- [3] D. Ruelle, *Chaos and Chance*, Princeton University Press, Princeton, 1991 (Good popular book).
- [4] D. Ruelle, Strange attractors, *Mathematical Intelligencer* 2 (1979) 126–137 (Good article with a few equations).
- [5] I. Stewart, *Does God Play Dice?: The Mathematics of Chaos*, Blackwell, Cambridge, MA, 1990 (Very good popular book).
- [6] S. Strogatz, *Nonlinear dynamics and Chaos : With Applications to Physics, Biology, Chemistry, and Engineering*, Addison-Wesley Pub., Reading, Mass., 1994 (Math book with lots of ideas but not many proofs).
- [7] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York, 1983 (Important math book with all the basic ideas and some heavy proofs).
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