

**ERRATA OF “LOCAL MONOMIALIZATION AND  
FACTORIZATION OF MORPHISMS”**

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We list here errata from our paper “Local monomialization and factorization of morphisms” [5].

page 23, line 25 (first line of statement of Theorem 2.7): “local domain” should be “regular local ring”.

page 39, line 28: “ $f_{ij} \geq 0$  for all  $i, j$ ” should be “ $f_{ij} \geq 0$  for  $0 \leq i, j \leq s$ ”.

page 45, line 15: “ $\bar{T}''(1)$  has regular parameters” should be “ $\bar{T}(1)$  has regular parameters”.

page 49, line 26: “ $s \leq m \leq l$ ” should be “ $s \leq m \leq n$ ”.

page 58, lines 19-22: Replace “Set  $\sigma(i)$  to be the largest possible . . . . .  $\sigma(i) \geq s$ ” with “Set

$$\sigma(i) = \dim k(\bar{T}(i))[[\bar{z}_1(i), \dots, \bar{z}_m(i)]]/p_m(i)$$

where  $p_m(i)$  is defined in (53). We have  $\sigma(i+1) \leq \sigma(i)$  for  $0 \leq i \leq t-1$  in (45)” The inequality  $\sigma(i+1) \leq \sigma(i)$  is proven in Lemma 6.3 of [6].

page 62, line 8: “By (42)” should be “By (43) and (43)”.

page 67, line 2: “ $u(\tilde{y}_1(t), \dots, \tilde{y}_l(t))$ ” should be “ $\Sigma(\tilde{y}_1(t), \dots, \tilde{y}_l(t))$ ”.

page 73, line 1: “ $g_{ij} \geq 0$  for all  $i, j$ ” should be “ $g_{ij} \geq 0$  for  $0 \leq i, j \leq s$ ”.

page 73, line 2: “ $U''(\alpha)[N_1, \dots, N_s, N_r]$ ” should be “ $U''(\alpha)[N_1, \dots, N_s, N_r, \frac{1}{N_r}]$ ”.

page 73, line 6: After “in an algebraic closure of  $k(U(\alpha+1))$ ” insert “if  $g_{s+1, s+1} > 0$ .”

$$M_r - c_{\alpha+1} = \prod_{i=1}^{-g_{s+1, s+1}} (N_r^{-1} - \omega^i d_{\alpha+1}^{-1}) \beta_r$$

where  $\omega$  is a primitive  $-g_{s+1, s+1}$ -th root of unity if  $g_{s+1, s+1} < 0$ .”

page 73, line 7: “ $U''(\alpha)[N_1, \dots, N_s, N_r]$ ” should be “ $U''(\alpha)[N_1, \dots, N_s, N_r, \frac{1}{N_r}]$ ”.

page 93, line 3: “ $\Omega \in m(U''(0))^N$ ” should be “ $\Omega \in m(U(0))^N$ ”.

page 93, after line 7, insert:

“(5): Suppose that  $g = \bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s} \Sigma(\bar{y}_1, \dots, \bar{y}_l) + \Omega$  where  $\nu(\bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s}) > A$  and  $\Omega \in m(U(0))^N$  with  $N\nu(m(U(0))) > \nu(\bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s})$ . Then there exists a CRUTS along  $\nu$  as in the conclusions of Theorem 4.9 such that

$$g = \bar{y}_1(t')^{d_1(t')} \cdots \bar{y}_s(t')^{d_s(t')} \bar{\Sigma}(\bar{y}_1(t'), \dots, \bar{y}_l(t'))$$

where  $\nu(\bar{y}_1(t')^{d_1(t')} \cdots \bar{y}_s(t')^{d_s(t')}) > A$

page 95, lines 26-31: “Let  $G$  be  $\dots g \in k(c_0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]] [x_{l+1}]$ ” should be “Since

$$\bar{y}_i^d = \bar{x}_1^{f_{i1}} \cdots \bar{x}_s^{f_{is}} \phi_1^{-f_{i1}} \cdots \phi_s^{-f_{is}}$$

for  $1 \leq i \leq s$ , by Lemma 4.2, we can perform a MTS of type (M1) to get  $g' \in k(U''(0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]] [x_{l+1}])$ .

Let  $G$  be the Galois group of a Galois closure of  $k(U''(0))$  over  $k(c_0)$ . Since  $x_{l+1}$  is analytically independent of  $\bar{y}_1^d, \dots, \bar{y}_s^d, \bar{y}_{s+1}, \dots, \bar{y}_l$  (by Theorem 2.12) we can define

$$g = \prod_{\tau \in G} \tau(g')$$

where  $G$  acts on the coefficients of  $g'$ . We have  $g \in k(c_0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]] [x_{l+1}]$

page 115, line 5 of the statement of Theorem 5.3: “if  $m_V$  is the maximal ideal of  $V$  and  $p^* = m_V \cap S$ ” should be “if  $q$  is a prime ideal of  $V$  and  $p^* = q \cap S$ ”.

page 115, line 6 of the statement of Theorem 5.3: “segments” should be “isolated subgroups”.

page 121, line 17: “ $t > \max\{a_{ij}, g_{ij}(1)\}$ ” should be “ $t > \max\{a_{ij}, g_{ij}(r)\}$ ”.

page 122, lines 4-5: “if  $m_V$  is the maximal ideal of  $V$  and  $p^* = m_V \cap S$ ” should be “if  $q$  is a prime ideal of  $V$  and  $p^* = q \cap S$ ”.

page 122, line 14: “as in the proof of Theorem 1.10 (Chapter 7)” should be “as in the proof of Theorem 5.1”.

page 122, lines 19-20: “if  $m_V$  is the maximal ideal of  $V$  and  $p^* = m_V \cap S$ ” should be “if  $q$  is a prime ideal of  $V$  and  $p^* = q \cap S$ ”.

page 122, line 6 of the statement of Theorem 5.5: “segments” should be “isolated subgroups”.

page 6, line 31: After “Theorem 1.6” insert:

“There is however uncertainty about the result of [9] (c.f. [1], [2]). There is a local version of this result for morphisms of toric varieties which has been proven. We may use this result instead of [9] to obtain proofs of Theorems 1.9 and 1.10 below. Christensen [3] has proven local strong factorization of morphisms of toric 3-folds along a toric valuation. Using the language of toric geometry, Karu [8] has extended Christiansen’s result to prove this local result in all dimensions. A proof in the spirit of Christensen’s original proof, using only elementary properties of determinants, is given in [7].”

page 136, line 10: After "Theorem 7.1", insert: "A proof of Theorem 1.9 using Theorem 1.1 (Local Monomialization) and Local Strong Factorization of morphisms of toric varieties along a toric valuation ([3], [8], [7]) which is thus independent of [9] is given in Theorem 3.3 [7]. A proof of Theorem 7.1 which is independent of [9] is now immediate from this proof of Theorem 1.9"

**Errata of “Local monomialization of transcendental extensions”**

page 1525, line 9 " $\nu(f_{I_0}) > \nu(f_I)$ " should be " $\nu(f_I) > \nu(f_{I_0})$ ".

page 1525, line 10 " $\nu(g_{J_0}) > \nu(g_J)$ " should be " $\nu(g_J) > \nu(g_{J_0})$ ".

## REFERENCES

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