Sample problems for test II

1. G is a group. $\phi : G \to G$ is the function $\phi(x) = x^2$.
a. Prove that G is abelian if and only if ϕ is a group homomorphism.
b. Show that G is finite, ϕ is an isomorphism if and only if G is abelian of odd order.
c. Give an example of a non abelian group and show why ϕ is not a homomorphism.

2. G is a finite group. Let $G = \{x_1, \ldots x_n\}$. If $x = \pi_{i=1}^{n} x_i$, is it true that $x^2 = 1$?

3. Prove or Disporve:
a. A group of order 30 cannot have a subgroup of order 8.
b. There are no non abelian groups of order 24.
c. All groups of order 17 are isomorphic to one another.

4. H and M are two normal subgroups of G of order 14 and 15 respectively. Which of these must be true? Why?
(i) HM is a subgroup of G.
(ii) HM is an abelian group.
(iii) HM is a normal subgroup of G.

5. G is a group. H is a normal subgroup of G. a. Show that the subgroups of the factor group G/H are of the form T/H where T is a subgroup of G containing H.
b. Prove that T/H is normal in G/H if and only if T is normal in G.
c. Is it true that if G/H and H are abelian, then G must be abelian?
d. If G is not abelian, but G/H is abelian, what can you say about H?

6. Give examples of:
a. A cyclic group of order 10.
b. An infinite abelian group and an infinite non abelian group.
c. A group G and a subgroup H which is not normal in G.
d. Three non isomorphic groups of order 20.
e. A group G and a group H and a function from G to H which is not a group homomorphism.