Home work 3
Due March 2

1. a. Prove that a group with even number of elements must have an element of order 2.
b. Prove that a group with even number of elements must have an odd number of elements of order 2.

2. Prove that $f : G \to G$ given by $f(x) = x^{-1}$ is a homomorphism if and only if G is abelian.

3. G is an abelian group. A and B are two subgroups of G. Let $AB = \{ab | a \in A, b \in B\}$.
Show that AB is a subgroup of G.
Did you use the fact that G is abelian? Where? Is it true if G is not abelian? Is G abelian necessary for this to be true? Why? Give examples to illustrate your answers.

4. a. Show that a group of order 30 cannot have a subgroup of order 8.
b. give an example of a group of order 12 with no subgroups of order 6.

5. Prove that the intersection of normal subgroups of G is normal in G.

6. How many normal subgroups does D_n have? List them.
How many normal subgroups does S_3 have? List them.
How many normal subgroups does S_4 have? List them.