

Home work 5  
Due Monday, April 13

1. a. Show that if  $\sigma$  is a permutation then  
 $\sigma(1, 2, \dots, k)\sigma^{-1} = (\sigma(1), \sigma(2), \dots, \sigma(k))$   
b.  $K = \{(1), (1, 2)(3, 4), (1, 3)(2, 4), (1, 4)(2, 3)\}$  is a normal subgroup of  $S_4$  and a normal subgroup of  $A_4$ .
2. Find the highest possible order for an element in  $S_8$  and an element in  $A_9$ .
3. a. Show that the set  $Z^{n \times n}$  of all  $n \times n$  matrices over  $Z$  is a ring under matrix addition and matrix multiplication and it is a subring of  $\mathcal{R}^{n \times n}$ .  
b. Show that the set  $R^{n \times n}$  is a ring for any commutative ring  $R$ .  
c. How many elements are there in  $Z_4^{3 \times 3}$ ? How many of them are invertible?
4. Show that  $Z_n$  is a field if and only if  $n$  is prime.
5. Prove that every field is an integral domain but not conversely. Also is it true that every finite integral domain is a field? Justify your answers.
6. Find all invertible elements in  $Z_{24}$ . Do they form a group under multiplication? Is this group cyclic? Answer the same questions for  $Z_{11}$ .