

Last Homework
Due May 6

1. Problem 22, 25 on page 147
2. Let i denote the square root of -1 . $R = \mathbb{Z}[i]$ is the ring of Gaussian integers. Let $M = \{a + bi \mid a, b \in 3\mathbb{Z}\}$
 - a. Show that M is an ideal and is maximal in R .
 - b. Show that R/M is a field with 9 elements
3. Construct a field with 49 elements
4. Determine if the following are irreducible in the specified fields.
 - a. $x^3 - 3x + 3$ over the rational numbers.
 - b. $x^3 - 9$ over \mathbb{Z}_{13}
 - c. $x^4 + 2x^3 - x^2 + 1$ over \mathbb{Z}_5 .
5. Show that $\mathbb{Z}[x][y]$ is isomorphic to $\mathbb{Z}[x, y]$.