TEST I-sample

1.(30)a. State the principle of Mathematical Induction

b. State the Well ordering principle and a theorem (other than the mathematical induction) which is an application of the well ordering principle.

b. Use mathematical induction to prove that
$(Cos\theta + iSin\theta)^m = Cosm\theta + iSinm\theta$ for all integers m.

2.(25) a. Define: A group.

b. Determine if the following are groups: Give reasons;
i. The set of all 2x2 matrices with positive rational entries under multiplication.
ii. The set of all positive rationals under addition.
iii. $T = \{x; x \text{ is a real number and } x \neq -1\}$. For $a, b \in T, a_*b = a + b - 1$.

c. Give examples of: a group G ; a proper subgroup H of G and a proper subset T of G which is not a subgroup of G.

These examples should be different from the ones in b.

3.(14) a. Define: The Greatest common divisor , gcd , of two integers.

b. Use Euclidean algorithm to find the gcd of 170 and 84.

4.(16) State True or False: If true prove it, if false give an example to show that it is false:
a. If a and b are relatively prime and a divides bc then a divides c.
b. A set is infinite if and only if it has infinitely many subsets.

5.(15) Prove that if $f : A \to B$ and $g : B \to C$ are two one-one functions then the compositon $g \circ f$ is also one-one.

Is the converse true? Justify your answer.