

TEST I - sample

Show All Your Work. Answers must be properly justified to get full credit.

1. (14) a. Determine all values of p and q for which the following system will have
 (a.) Exactly one solution, (b.) No solution and (c.) Infinitely many solutions:

$$\begin{aligned} x + 2y - 2z &= 1 \\ 3x - y + pz &= 2q \\ 2x + y + z &= 3 \end{aligned}$$

- b. (12) Find all the solutions to:

$$\begin{bmatrix} x_1 + 2x_2 - 3x_3 + x_4 &= 2 \\ 4x_1 - x_2 + 2x_3 &= 3 \\ x_2 - 3x_4 &= 1 \end{bmatrix}$$

2. (20) Determine which of the following matrices are invertible. Find the inverse, if it exists.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 3 \\ 0 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 3 & 9 \\ 0 & 0 & 3 & 10 \\ 1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

3. (8) Give examples of
 a. Two matrices A and B such that AB is not defined.
 b. A 2×2 matrix of determinant 6.
 3. (10) .

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -2 & 0 \\ 2 & 1 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 1 \\ 4 & -3 & 4 \end{bmatrix}$$

Determinant of $A = 23$

What is the determinant of B ?

What is the determinant of C ?

What is the determinant of $2A = A + A$?

4. (28) Short answers:

a. The reduced row echelon form of a 4×4 matrix B has a row of zeros. Is it true that the reduced row echelon form of AB also has a row of zeros? Why?

b. Write three conditions on a square matrix that are equivalent to saying it is invertible.

c. $A = \{a_{ij}\}$ is a 4×4 matrix such that $a_{ij} = i^2 - 3ij + 2j$. Write down A and compute A^2

d. What is an elementary matrix? Write down the 4×4 elementary matrix E corresponding to the row operation of adding 2 times the 4th row to the third row.

- 5 (8) Just state True or false:

1. Elementary matrices are invertible but not always symmetric.
2. If A and B are invertible $m \times m$ matrices, then so is $A + B$.
3. If A has a row of zeros then the determinant of A is zero.

4. If the reduced row echelon form of A is I , then the reduced row echelon form of $3A$ is $3I$.