Matrix theory
Home-work 4
Maximum 25 points
Due October 14

1. (6 points) Determine if the following are vector spaces:

(i) The set of all $n \times n$ symmetric matrices under usual matrix addition and multiplication by real numbers.

(ii) The set of all 4×4 invertible matrices under usual matrix addition and multiplication by real numbers.

(iii) The set of all solutions to a system of m linear homogeneous equations in n variables.

2. (4 points) Is $\{p(x)|p(2) = p(5), p \in P_{100}\}$, the set of all polynomials $p(x)$ in P_{100} satisfying $p(2) = p(5)$ a subspace of P_{100}? Justify your answer.

3. (4 points) $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are linearly independent vectors in a vector space V. Determine if $\{v_1 + v_2, v_2 - v_3, v_1 - v_3 - v_4, v_2 + v_4\}$ is linearly independent.

4. (4 points) a. Find a basis for the subspace S of all diagonal 3×3 matrices of $R^{3\times 3}$.

b. Find a basis for the subspace $S = \{ \begin{bmatrix} a \\ 2a-b \\ a+c \\ 4a-b-c \end{bmatrix} |a, b, c \text{real numbers}\}$ of R^4. What is its dimension?

5. (4 points) Determine if x^2+4x-3 in the span of $\{x^3-2x+1, x^3+2x^2+3, x^2-4x\}$.

6. (3 poins) Write two non trivial subspaces V and W of R^5 of dimension 3. What is $V \cap W$, the intersection of V and W. (Recall that $V \cap W$ is the set of all elements common to both V and W.)

(Your answer will depend on your examples for S and T).