Homework V
First draft due October 21

1. Let S^n be the vector space of all $n \times n$ symmetric matrices.
a. Find a basis for S^3

b. What is the dimension of S^3?

c. Is there a basis for S^3 all of whose vectors are invertible? If yes, find one. If not, explain (prove) why not.

d. Is there a basis for S^3 all of whose vectors are singular? If yes, find one. If not, explain (prove) why not.

e. Is there a basis for $R^{3\times 3}$ all of whose vectors are symmetric? If yes, find one. If not, explain (prove) why not.
f. Is it true that S^8 is isomorphic to $R^{n\times n}$ for some n? How about S^5? Justify your answers.

2. Find the transition matrix (change of basis matrix)from the basis $\{x^2 - 3x + 1, 2x - 3, 2\}$ to the basis $\{2 + 5x + 7x^2, 5 + 14x, 14\}$.

3. $\mathbf{v_1}, \mathbf{v_2} \cdots \mathbf{v_n}$ are linearly independent vectors in a vector space V. $\mathbf{v}$ is a vector in V. Prove that
a. If $\mathbf{v}$ is in the span of $\mathbf{v_1}, \mathbf{v_2} \cdots \mathbf{v_n}$, then $\mathbf{v}, \mathbf{v_1}, \mathbf{v_2} \cdots \mathbf{v_n}$ are not linearly independent.
and
b . If $\mathbf{v}$ is not in the span of $\mathbf{v_1}, \mathbf{v_2} \cdots \mathbf{v_n}$ then $\mathbf{v}, \mathbf{v_1}, \mathbf{v_2} \cdots \mathbf{v_n}$ are linearly independent .

4. a. Find the coordinate vector of

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$$

in the basis

$$B = \{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \}$$

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b. What are the coordinates of $2A$ and A^2? Explain your reasoning.

5. Find the matrix that represents L in the basis $B = \{1 - x, 1 - 2x + x^2, 2 + x\}$ of P_3.
$L : P_3 \to P_3$ is given by $L(f(x)) = f(1) + 2xf(-1) + x^2 f(2)$.

6. $L : V \to W$ is a linear transformation. $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$ are vectors in V. $\mathbf{w_i} = L(\mathbf{v_i})$. Show that if $\mathbf{w_1}, \mathbf{w_2}, \ldots \mathbf{w_n}$ are linearly independent, then $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$ are also linearly independent.