Home work 6
Due November 13

1. a. V is an inner product space with an inner product $*$. $\mathbf{v}$ is a vector in V. Show that $\mathbf{v}^{\perp} = \{\mathbf{w} \in V | \mathbf{w} * \mathbf{v} = 0\}$ is a subspace of V.

b. If $V = P_4$ and $f * g = f(-2)g(-2) + f(1)g(1) + f(-1)g(-1) + f(2)g(2)$, find $(x^2 + x + 1)^{\perp}$.

2. $V = R^3$, $(a_1, b_1, c_1)^T * (a_2, b_2, c_2) = 2a_1a_2 + 3b_1b_2 + c_1c_2$
a. What is the length of $(2, 1, 1)^T$?
b. Is $(1, -1, 1)^T$ orthogonal to $(2, 3, 1)^T$? Why?

3. $V = R^4$ with dot product.
a. $S = Span\{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ -1 \end{bmatrix}\}$ Find a basis and compute the dimension of $S^{\perp}$.

b. Convert to an orthonormal basis using Gram-Schmidt algorithm.

$$\{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ -1 \end{bmatrix}\}.$$

4. A matrix A is called orthogonal if its columns form an ortho-normal basis for R^n with dot product. Give an example of an orthogonal matrix (2×2 or 3×3) and an invertible non orthogonal matrix. If A is an orthogonal matrix, is $adjA$ also an orthogonal matrix? Why?

Home work 7a

first draft Due November 11

1. a. Prove that trace (AB) = trace (BA) for any two $n \times n$ matrices. b. Consider $R^{n\times n}$, with $A * B = Trace(AB)$. Determine if $*$ defines an inner product on $R^{2\times 2}$.

If yes, find an orthonormal basis for $R^{2\times 2}$ under this inner product.

If not, explain why not and give an example to prove it.

c. Consider $R^{n\times n}$ with $A <> B = Trace(AB^T)$

Determine if $<>$ defines an inner product. If yes, find find an orthonormal basis for $R^{2\times 2}$ for this product. If not, explain whynot and give an example to prove it.

2. Consider the inner product space P_3 with the inner product, $F * G = f(1)g(1) + f(2)g(2) + f(-4)g(-4)$.

a. Find the length of x and x^2.

b. Is x orthogonal to x^2?

c. Find a non zero vector in P_3 that is orthogonal ot x in this product.

d. Choose any basis $\{f_1, f_2, f_3\}$ of P_3. Form the 3×3 matrix $A = (f_i * f_j)$.

e. What can you say about your matrix in d . Is it symmetric? Is it invertible? Is it diagonal? Is it upper trianglular?

f. Without discussing with your classmate (of your choice, name the student), say what his or her correct answer to e would be.

g. Check with your class mate and report how far you agreed. Can you explain the agreements and the disagreements?