

Home work 7a  
first draft Due November 11

1. a. Prove that  $\text{trace}(AB) = \text{trace}(BA)$  for any two  $n \times n$  matrices. b. Consider  $R^{n \times n}$ , with  $A * B = \text{Trace}(AB)$ . Determine if  $*$  defines an inner product on  $R^{2 \times 2}$ .  
If yes, find an orthonormal basis for  $R^{2 \times 2}$  under this inner product.  
If not, explain why not and give an example to prove it.  
c. Consider  $R^{n \times n}$  with  $A \langle \rangle B = \text{Trace}(AB^T)$   
Determine if  $\langle \rangle$  defines an inner product. If yes, find an orthonormal basis for  $R^{2 \times 2}$  for this product. If not, explain why not and give an example to prove it.
  
2. Consider the inner product space  $P_3$  with the inner product,  $F * G = f(1)g(1) + f(2)g(2) + f(-4)g(-4)$ .
  - a. Find the length of  $x$  and  $x^2$ .
  - b. Is  $x$  orthogonal to  $x^2$ ?
  - c. Find a non zero vector in  $P_3$  that is orthogonal to  $x$  in this product.
  
  - d. Choose any basis  $\{f_1, f_2, f_3\}$  of  $P_3$ . Form the  $3 \times 3$  matrix  $A = (f_i * f_j)$ .
  - e. What can you say about your matrix in d. Is it symmetric? Is it invertible? Is it diagonal? Is it upper triangular?
  - f. Without discussing with your classmate (of your choice, name the student), say what his or her correct answer to e would be.
  - g. Check with your class mate and report how far you agreed. Can you explain the agreements and the disagreements?