Test II SAMPLE

1. a. Define: A basis:

b. Find a basis and compute the dimension of the column space of the following matrix. What is the rank of this matrix.

$$\begin{bmatrix} 1 & -1 & 4 & 0 & 3 \\ 2 & 1 & 3 & 1 & 2 \\ 4 & 5 & 1 & 3 & 0 \end{bmatrix}$$

2. a. T is the set of all 3×3 upper triangular matrices. Is T a vector space under usual additiona nd scalar multiplication? Justify your answer.

b. Determine if the following set of vectors is linearly independent in $R^{2\times 2}$.

$$\{\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 1 & 2 \end{pmatrix}\}$$

c. If B is a 4×4 matrix of determinant 2 what is the rank of $3B$?

3. a. Give examples of: (you must justify your examples).
i. A Linear transformation $T : R^3 \to P_4$

ii. Two non zero matrices with the same row space.

b. $L : R^{2\times 2} \to P_3$ is a linear transformation which takes the identity matrix to 0 and $L(\begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix}) = x$. Can you tell what is $L(\begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix})$? Can you tell what is $L(\begin{matrix} 0 & 0 \\ 1 & -1 \end{matrix})$?

4. $L : P_4 \to P_3$ is given by $L(f(x) = f(1)x^2 + f(-1)x + 2f(0)$.
a. Find the matrix representing the linear transformation L in the basis $\{x^3, x^2 + 2x - 1, x + 5, 4\}$ of P_4 and the basis $\{3x - 1, x + 2, x^2 - 1\}$ of P_3

b. Find a basis for the Kernel of L and a basis for the Image of L.

c. What is the rank of L.

5.. a) Define: Dimension of a vector space:

b. $T : V \to W$ is a linear transformation. Suppose T is one-one. Prove that if the vectors $\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}$ are linearly independent in V, then $T(\mathbf{v_1}), T(\mathbf{v_2}), \cdots T(\mathbf{v_n})$ must be linearly independent vectors in W. Clearly identify the place you use the fact T is one-one. Give an example to show that this is not true if T is not one-one.

Alternate: $T : V \to W$ is a linear transformation. Prove that if the vectors $\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}$ are such that their images $T(\mathbf{v_1}), T(\mathbf{v_2}), \cdots T(\mathbf{v_n})$ are linearly independent vectors in W. Prove that $\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}$ must be linearly independent. Is the converse true? Justify your answer.

6. a. Find the transition matrix from the basis $B = \{\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}\}$ of R^3 to the basis $C = \{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}\}$.

b. The coordinate of $\mathbf{v}$ in the basis $B = \{x, x + 1, x^2 + 1\}$ of P_3 is $X_B(\mathbf{v}) = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

Find the vector $\mathbf{v}$