Quiz. *4*

Compute the limits

(a) $\lim_{x\to 1^-} \frac{2-3x}{x-1}$,
(b) $\lim_{x\to 0^-} \frac{1}{x^2-2x}$,
(c) $\lim_{x\to\infty} \frac{x^7-35x^3+8}{3x^7+1}$,
(d) $\lim_{x\to-\infty} \frac{x^11+4x^2+9}{x^9-36}$.

Solution

(a): When $x \to 1^-$ the denominator is "going" to 0, while the numerator gives a non-zero number, -1. Hence, the limit will be infinity. Now, we need to determine whether it is positive or negative infinity. We think of a number very close to 1 from the right, say 0.9. Then, the numerator is negative $(2 - 3 \cdot 0.9 < 0)$ and the denominator is positive $(0.9 - 1 < 0)$. Hence,

$$\lim_{x\to 1^+} \frac{2-3x}{x-1} = +\infty.$$

Note that, if instead of 0.9 we had chosen a number which is not very close to 1, say, 0.5, then the answer would had been different, since $2 - 3 \cdot 0.5 > 0$!!

(b): It is usually best to factor out the quantities:

$$\lim_{x\to 0^-} \frac{1}{x^2-2x} = \lim_{x\to 0^-} \frac{1}{x(x-2)} = +\infty$$

For the next two limits we see the "long" solution. You don't necessarily need to use it, but it is always good to know it.

(c):

$$\lim_{x\to\infty} \frac{x^7-35x^3+8}{3x^7+1} = \lim_{x\to\infty} \frac{x^7(1-\frac{35}{x^4}+\frac{8}{x^7})}{x^7(3+\frac{1}{x^7})}$$
$$= \lim_{x\to\infty} \frac{(1-\frac{35}{x^4}+\frac{8}{x^7})}{(3+\frac{1}{x^7})} = \frac{1}{3},$$

since when $x \to \infty$, each one of the terms: $\frac{35}{x^4}$, $\frac{8}{x^7}$, $\frac{1}{x^7} \longrightarrow 0$.

(d): Similarly, as in the previous example:

$$\lim_{x\to-\infty} \frac{x^{11}+4x^2+9}{x^9-36} = \lim_{x\to-\infty} \frac{x^{11}(1+\frac{4}{x^9}+\frac{9}{x^11})}{x^9(1-\frac{36}{x^9})}$$

$$= \lim_{x \to -\infty} \frac{x^2(1 + \frac{4}{x^9} + \frac{9}{x^11})}{1 - \frac{36}{x^9}} = +\infty.$$

Note that, even though $x \to -\infty$, the limit "goes" to $+\infty$, because $x^2 \to \infty$ and everything else is positive.