

Amortization of Loans

1. To find the principal (or amount) of the loan, use the formula for the present value of a decreasing annuity:

$$P = \frac{(1+i)^n - 1}{i(1+i)^n} \cdot R,$$

where P is the principal of the loan, i is the interest rate per period, n is the total number of interest periods (which is equal to the number of payments), R is the amount paid back at the end of each interest period.

2. To find the unpaid balance of the loan after K interest periods, first calculate the number N of interest periods left:

$$N = [\text{total number of interest periods}] - K.$$

Then use the formula for the present value of a decreasing annuity with the number N instead of n :

$$[\text{unpaid balance}] = \frac{(1+i)^N - 1}{i(1+i)^N} \cdot R.$$

3. To find the amount of principal paid off during the L -th year of the loan, calculate the unpaid balance after $(L - 1)$ years (see part 2), calculate the unpaid balance after L years (see part 2 again), and subtract the second number from the first one.

Note: when using the formula for the unpaid balance from part 2, you need to use the number of interest periods, not years.

4. To find the amount of interest paid during the L -th year of the loan, first calculate the amount of principal paid off during the L -th year of the loan (see part 3), then calculate the total amount paid during the L -th year (equal to the amount R paid at the end of each interest period multiplied by the number of payments per year), and subtract the first number from the second.

5. Balloon payments:

$$[\text{Amount of the loan}] =$$

$$[\text{Present value of annuity payments}] + [\text{Present value of balloon payment}]$$

So, to find the amount of the loan which includes regular annuity payments and the balloon payment at the end of the term, you can use the formula:

$$[\text{Amount of the loan}] = \frac{(1+i)^n - 1}{i(1+i)^n} \cdot R + \frac{B}{(1+i)^n},$$

where B is the amount of the balloon payment, for the rest of the notations see part 1.