Real Analysis II

Questions for Exam 1

Approximation of continuous functions

- Proof of Weierstrass theorem via Bernstein polynomials.
- Stone–Weirstrass theorem for real-valued functions. **(proof)**
- Stone–Weirstrass theorem for complex-valued functions.

L_p spaces

- Holder inequality. **(proof)**
- Minkowski inequality. **(proof)**
- Jensen inequality.
- Completeness of the L_p-spaces.
- General form of a linear functional on the space L_p. **(proof)**
- Integral Minkowski inequality. **(proof)**
- Distribution function formula.
- Hardy–Littlewood maximal theorem for $L_p,\ p > 1$. **(proof)**
- Elementary properties of convolution.
- Convergence of the convolution of a function with an approximate identity to the function itself in L_p.
- Almost everywhere convergence of the convolution of a function with an approximate identity to the function itself.
- Rademacher theorem.

Hilbert space.

- Definition of a Hilbert space.
- Cauchy-Schwartz inequality and parallelogram equality.
- Existence of a metric projection on a closed convex set. (**proof**)
- Metric projection is a contraction.
- Orthogonal projection on a subspace.
- Muntz theorem in $L_2([0,1])$.
- Muntz theorem in $C([0,1])$. (**proof**)
- Riesz representation theorem. (**proof**)
- Hilbert space proof of Lebesgue–Radon–Nikodim theorem.
- Bessel's inequality.
- Riesz-Fischer theorem.
- A necessary and sufficient condition for an orthonormal system to be a basis (Parseval equality). (**proof**)

- Haar system.
- Walsh system.
- Khinchinne's inequality.