

Real Analysis II

Questions for Exam 2

- Equidistributed sequences. Weyl's theorem.
- Riemann–Lebesgue lemma on $[0, 2\pi]$.
- Dirichlet kernel.
- Localization principle.
- Dini's theorem. (**proof**)
- Fejer kernel.
- Fejer-Lebesgue Theorem for continuous functions. (**proof**)
- Fejer-Lebesgue Theorem for L_1 functions.
- Definition of the Fourier transform on \mathbb{R} .
- Riemann–Lebesgue lemma on \mathbb{R} .
- Plancherel theorem. (**proof**)
- Hermite functions form a basis of $L_2(\mathbb{R})$.
- Fourier inversion formula. (**proof**)
- Sobolev Theorem. (**proof**)
- Schwartz space. Definition of the topology.
- Fourier transform is one-to-one and onto on the Schwartz space.
- The space of tempered distributions.
- Differentiation, multiplication by x^k , and Fourier transform of distributions.
- Distributions with compact support.
- If F is a distribution with compact support, f is a $C^\infty(\mathbb{R}^n)$ function with compact support, disjoint from the support of F , then $\langle F, f \rangle = 0$.
- The Fourier transform of a distribution with compact support is a slowly increasing function. Formula for the Fourier transform of a distribution with compact support. (**proof**)
- Any distribution with compact support is a certain derivative of a continuous function, which is 0 at ∞ .
- Paley–Wiener Theory.
 - Fourier transform of a function analytic in the strip. (**proof**)
 - Fourier transform of a function analytic in the half-plane.
 - Fourier transform of an entire function of exponential growth. (**proof**)
- Hausdorff measure and Hausdorff dimension.
- Existence of a supporting measure for a set of a positive Hausdorff measure.