

# Interpolation estimates of the Carleson maximal operators and the multiple Fourier series

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**Abstract.** We prove the novel interpolation estimates for the Carleson maximal operators generated by the partial sums of the multiple Fourier series and all its conjugate series over cubes defined on the  $d$ -dimensional torus  $\mathbb{T}^d$ . We show that these operators are bounded from a variant of the Arias-de-Reyna space  $QA^d$  to the weak  $L^1$ -space on  $\mathbb{T}^d$ . This implies that the multiple Fourier series of every function  $f \in QA^d$  and all its conjugate series converge over cubes almost everywhere. By a close analysis of the space  $QA^d$  we prove that it contains a Lorentz space that strictly contains the Orlicz space  $L(\log L)^d \log \log \log L(\mathbb{T}^d)$ . As a consequence we obtain an improvement of a deep theorem proved by Antonov which was the best known result on the convergence of multiple Fourier series over cubes. The talk is based on the joint work with L. Rodríguez-Piazza.