The Gaussian measure of thin shells and the randomized Dvoretzky's theorem

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Abstract

We prove the following two-sided inequality for the standard Gaussian measure γ_n in $\mathbb{R}^n$ and all $\varepsilon \in (0, 1/2]$:

$$1 - 2n^{-c\varepsilon/\ln\frac{1}{\varepsilon}} \leq \inf_K \sup_T \gamma_n\big((T(K) + \varepsilon T(K)) \setminus T(K)\big) \leq 1 - n^{-C\varepsilon}.$$

Here, supremum is taken over all invertible linear transformations $T : \mathbb{R}^n \to \mathbb{R}^n$ and the infimum — over all compact centrally-symmetric convex sets $K \subset \mathbb{R}^n$ with non-impty interior; c and C are universal constants. The inequalities have found interesting applications to the problem of dependence on distortion ε in the "randomized" Dvoretzky's theorem. Namely, we derive as a corollary that for any K, there is an operator T such that the measure of $\lfloor \tilde{c}\varepsilon \ln n / \ln^2 \frac{1}{\varepsilon} \rfloor$-dimensional $(1+\varepsilon)$-spherical sections of $T(K)$ is at least $1 - 2n^{-\tilde{c}\varepsilon/\ln\frac{1}{\varepsilon}}$. On the other hand, for the unit cube B_∞^n and all $\varepsilon \ll (\ln n)^{-1}$ and $k \geq 2$, there is no operator T such that most k-dimensional sections of $T(B_\infty^n)$ are $(1+\varepsilon)$-spherical. This result reveals a principal difference between the "randomized" setting and the classical problem of existence of almost Euclidean sections, as far as dependence on ε is concerned.