Marginals of log-concave measures with bounded isotropic constant

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Abstract

A convex body $K \subset \mathbb{R}^n$ is isotropic if its volume is one, its center of mass is the origin and there is a constant $L_K > 0$ (called the isotropic constant) such that

$$\int_K \langle x, \theta \rangle^2 dx = L_K^2$$

for each unit vector θ. It is unknown whether or not L_K admits a uniform bound, independent of the body K and the dimension n. This is known as the Isotropic Constant Conjecture or, due to one of its many equivalent formulations, the Slicing Problem. Currently, the best known result is $L_K \leq Cn^{1/4}$, proved by Klartag, improving on an earlier result of Bourgain. Klartag, in fact, showed that given any convex body K, there is another body "close" to K, which has a bounded isotropic constant, and thus "isomorphically" the conjecture is true.

There is an equivalent version of the Slicing Problem in terms of log-concave probability measures and the isotropic constants of their marginal distributions. In this talk, I will explain why all such measures admit k-dimensional marginals ($k \leqslant \sqrt{n}$) with bounded isotropic constants and these are plentiful: they form a dense set on the Grassmannian of all k-dimensional linear subspaces of $\mathbb{R}^n$. This is joint work with Grigoris Paouris.