More on logarithmic sums of convex bodies

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Abstract

We prove that the log-Brunn-Minkowski inequality (log-BMI) for the Lebesgue measure in dimension n would imply the log-BMI and, therefore, the B-conjecture for any log-concave density in dimension n. As a consequence, we prove the log-BMI and the B-conjecture for any log-concave density, in the plane. Moreover, we prove that the log-BMI reduces to the following: For each dimension n, there is a density f_n, which satisfies an integrability assumption, so that the log-BMI holds for parallelepipeds with parallel facets, for the density f_n. As byproduct of our methods, we study possible log-concavity of the function $t \mapsto |(K+_p \cdot e^t L)^\circ|$, where $p \geq 1$ and K, L are symmetric convex bodies, which we are able to prove in some instances and as a further application, we confirm the variance conjecture in a special class of convex bodies. Finally, we establish a non-trivial dual form of the log-BMI.