More on logarithmic sums of convex bodies

Christos Saroglou (Texas A&M University)

Abstract

We prove that the log-Brunn-Minkowski inequality (log-BMI) for the Lebesgue measure in dimension $n$ would imply the log-BMI and, therefore, the B-conjecture for any log-concave density in dimension $n$. As a consequence, we prove the log-BMI and the B-conjecture for any log-concave density, in the plane. Moreover, we prove that the log-BMI reduces to the following: For each dimension $n$, there is a density $f_n$, which satisfies an integrability assumption, so that the log-BMI holds for parallelepipeds with parallel facets, for the density $f_n$. As byproduct of our methods, we study possible log-concavity of the function $t \mapsto \| (K + p \cdot e^t L)^\circ \|$, where $p \geq 1$ and $K, L$ are symmetric convex bodies, which we are able to prove in some instances and as a further application, we confirm the variance conjecture in a special class of convex bodies. Finally, we establish a non-trivial dual form of the log-BMI.