On the shape of a convex body with respect to its second projection body

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Abstract

We develop a new approach to the problem of finding sharp bounds for the affine invariant $P(K) = V(\Pi K)/V^{d-1}(K)$. Namely, we prove that if $K$ is a 3-dimensional zonoid of volume 1, then its second projection body $\Pi^2 K$ is contained in $8K$, while if $K$ is any symmetric 3-dimensional convex body of volume 1, then $\Pi^2 K$ contains $6K$. Both inclusions are sharp. Consequences of these results include a stronger version of a reverse isoperimetric inequality for 3-dimensional zonoids-established by the author in a previous work, a reduction for the 3-dimensional Petty conjecture to another isoperimetric problem and the best known lower bound up to date for $P(K)$ in 3 dimensions. As byproduct of our methods, we establish an almost optimal lower bound for high-dimensional bodies of revolution.