Randomized versions of isoperimetric inequalities

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Abstract

The Blaschke-Santaló inequality is among the most fundamental affine isoperimetric inequalities in convex geometry. It asserts that for a convex body $K \subset \mathbb{R}^n$ with polar dual $K^\circ := \{x \in K : \langle x, y \rangle \leq 1 \forall y \in K\}$, the *volume product*

$$p(K) = \mathrm{vol}(K)\,\mathrm{vol}(K^\circ)$$

is maximized precisely when K is an ellipsoid. Among convex bodies that are centrally-symmetric, i.e., $K = -K$, a minimizer of $p(K)$ is conjectured to be the cube $Q = [-1, 1]^n$, known as Mahler's conjecture, which has remained open for $n \geq 3$ since the 1940s. It is known to be true in an asymptotic sense, i.e., $p(K)^{1/n} \geq cp(Q)^{1/n}$, first proved by [Bourgain-Milman, 1987], with remarkable new proofs by [Kuperberg, 2008], [Nazaraov, 2010], and [Giannopoulos-Vritsiou-Paouris, 2012].

I will discuss a randomized version of the Blaschke-Santaló inequality. This gives rise to a new family of inequalities in which the extremal sets are not necessarily ellipsoids but random sets that one generates by using ellipsoids. The new randomized versions interpolate between several such inequalities, with the classical versions appearing via the law of large numbers. The talk will be part expository and partly based on recent joint work with D. Cordero-Erausquin, M. Fradelizi and G. Paouris.