Steady concentrated vorticity of the Euler equation in 2-d domains

Chongchun Zeng
Georgia Tech
chongchun.zeng@math.gatech.edu

Abstract

In this talk, we consider steady solutions of the incompressible Euler equation in 2-dim domains with concentrated vorticity. In the case of a fixed domain $\Omega \subset \mathbf{R}^2$, with prescribed integer $n > 0$, small vortical domain sizes $r_1, \ldots, r_n > 0$, and vorticity strengths $\mu_1, \ldots, \mu_n \neq 0$, we seek steady vorticity distributions in the form of

$$\omega = \sum_{j=1}^{n} \omega_j(x)$$

where

1.) the vortical domains satisfy $\Omega_j := supp(\omega_j) \subset B(p_j, 2r_j)$, $|\Omega_j| = \pi r_j^2$, with distinct $p_1, \ldots, p_n \in \Omega$; and

2.) $\mu_j = \int \omega_j dx$.

Since the dynamics of localized vorticity is approximated by the point vortex dynamics, we take $\{p_1, \ldots, p_n\}$ close to a non-degenerate steady configuration of the point vortex system in Ω with parameters $\mu_1, \ldots, \mu_n$. Through a perturbation method applied to Ω_j parametrized by conformal mappings, we obtained two types of steady solutions with smooth $\partial\Omega_j$ being $O(|r|r_j^2)$ perturbations to $\partial B(p_j, r_j)$: a.) infinitely many piecewise smooth solutions $\omega \in C^{0,1}(\Omega)$; and b.) a unique steady vortex patch with piecewise constant vorticity, i.e.

$$\omega_j = \frac{\mu_j}{\pi r_j^2} \chi(\Omega_j).$$

Moreover, the spectral and evolution properties (stability, exponential trichotomy, etc.) of the linearized vortex patch dynamics at the latter is largely determined by those of the linearized point vortex dynamics at the steady configuration $\{p_1, \ldots, p_n\}$. Steady 2-dim gravity-capillary water waves (free boundary problem) will be also discussed.