Valuations on Function Spaces

Monika Ludwig

Let $F$ be a space of real valued functions, for example, the Sobolev space $W^{1,1}(\mathbb{R}^n)$ and let $A$ be an abelian semi-group. A function $z : F \rightarrow A$ is called a valuation if

$$z(f \vee g) + z(f \wedge g) = z(f) + z(g)$$

for all $f, g \in F$, where $f \vee g$ denotes the pointwise maximum and $f \wedge g$ the pointwise minimum of $f$ and $g$.

Valuations are a classical subject in the theory of convex bodies and the theory of valuations has seen a rapid development in recent years. We give a short overview of pertinent results.

We also give a complete classification of affinely contravariant convex body valued valuations on $W^{1,1}(\mathbb{R}^n)$. We show that there is a unique such valuation, which turns out to be closely related to the optimal Sobolev body introduced by Lutwak, Yang & Zhang.