A priori symmetry and decay properties of a non-local shallow water wave equation

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Abstract. In this talk, we prove the following interesting inter-connection: that traveling solitary waves of a non-local wave equation are necessarily symmetric, monotone on a half-line, and of exponential decay rate; and that symmetric solutions of the initial-value problem for the same equation are necessarily traveling. Whereas the second proof is based on a quite general structural property (which we extend here to a non-local setting), the proof of the first three facts relies on a detailed analysis of the Fourier transform of $m(\xi) = \sqrt{\tanh \xi}$, which we prove is completely monotone. More precisely, we study the Whitham equation

$$u_t + uu_x + \int K(x-y)u_x \, dx = 0,$$

where the integral kernel $K$ has the symbol $m(\xi)$ above. This symbol, and its action, is an inherent part in the study of water waves.