Exact controllability and Stabilization of solutions by feedback control for Navier-Stokes equations

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Abstract. Two control problems for the Navier-Stokes equations will be considered.

First, the exact controllability problem: Given initial condition $v_0(x)$ and a restriction $v(T, x)$ of some solution at prescribed time $T > 0$, find control function $u(t, x)$ such that the solution of the controlled Navier-Stokes equations from $v_0(x)$ coincides with $v(T, x)$ at time $t = T$. To solve this problem a distributed control is used, i.e. a vector field $u(t, x)$ on the right side of the system, supported in an arbitrary fixed subdomain of the space domain where the Navier-Stokes system is defined.

In addition, the problem of stabilization of solution by feedback control will be considered. In this problem, instead of $v(T, x)$, a steady-state solution $w(x)$ is given, and one should choose such control that the solution from $v_0(x)$ tends to $w(x)$ exponentially as the time $t$ tends to infinity. The key point here is that the control $u$ should be feedback, i.e. it can react to unpredictable fluctuations of the solution by dumping them.

The main ideas of the solution to the problems above will be discussed.