Representation Theorems for indefinite quadratic forms and applications

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Abstract. One of the main problems in the operator theory is to give necessary and sufficient conditions for a symmetric sesquilinear form $b$ to uniquely define a self-adjoint operator $B$ such that

$$b[x, y] = (x, By), \ x \in \text{Dom}[b], y \in \text{Dom}(B) \ (\text{First Representation Theorem}).$$

The converse problem is to reconstruct the form $b$ from the operator $B$. For the reconstruction, it is important to know whether the domain stability property holds, that is, $\text{Dom}(b) = \text{Dom}(|B|^{1/2})$, and if

$$b[x, y] = (|B|^{1/2}x, \text{sign}(B)|B|^{1/2}y) \ (\text{Second Representation Theorem})$$

indeed agrees with $b$.

For bounded or semibounded closed forms, classic results give a complete solution to these problems.

In this talk we address these problems for indefinite, not necessarily semibounded forms.

As an application of the representation theorems, we study non-elliptic differential operators of the type $\text{div} \ H \text{grad}$ that attracted some interest in the context of optical meta-materials and their relation to cloaking phenomena.

We also revisit the semibounded situation, in particular the Stokes operator, extending known results to allow unbounded domains. For the Stokes operator considered as a block operator matrix, we provide an explicit block diagonalization that allows a closer inspection of the spectrum.

This talk is based on the speakers PhD Thesis and also on joint work with A. Hussein, V. Kostrykin, D. Krejčičík, K. A. Makarov, and K. Veselić.