Sequences of Rank-1 Projections and Gabor Tight Fusion Frames

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ABSTRACT

This dissertation provides new results in two different areas. The first concerns properties inherited by sequences of orthogonal rank-1 projections (ie, the outer product sequences such as \( \{f_i f_i^* \}_{i=1}^M \)) within the Hilbert space of symmetric operators \( \text{sym}(\mathcal{H}^{N \times N}) \) from their inducing unit-norm vector sequences \( \{f_i\}_{i=1}^M \) within a Hilbert space \( \mathcal{H}^N \); notably, we show the cases where quantitative Riesz and frame bounds of \( \{f_i\}_{i=1}^M \) are inherited by the induced projections \( \{f_i f_i^*\}_{i=1}^M \). We then show that the family of unit norm frames which yield independent outer product sequences is open and dense (in a Euclidean-analytic sense) within the topological space \( \bigotimes_{i=1}^M S_{N^1} \), where \( M \leq \text{dim} \text{sym}(\mathcal{H}^{N \times N}) \). We then give a full geometric characterization of the particular sequences that produce dependent sequences of projections.

The second part concerns a new method to construct so-called tight fusion frames. Hilbert space fusion frames are a natural extension of Hilbert space frames, extending the notion from a set of vectors in a Hilbert space to a set of subspaces of a Hilbert space with analogous notions of overcompleteness and boundedness. As tight frames are a very important topic within standard frame theory, tight fusion frames are similarly important; however, only trivial examples of tight fusion frames are hitherto known. Here we apply ideas from Gabor analysis to demonstrate a non-trivial construction of tight fusion frames. We then use this construction to further show their applicability in some cases for the retrieval of signals modulo phase.