Center manifolds without a phase space for quasilinear PDE from elasticity, biology, and hydrodynamics

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Abstract

In this talk, we present a new center manifold reduction theorem for quasilinear elliptic equations posed on infinite cylinders. This is done without a phase space in the sense that we avoid explicitly reformulating the PDE as an evolution problem. Under suitable hypotheses, the resulting center manifold is finite dimensional and captures all sufficiently small bounded solutions. Compared with classical methods, the reduced equation on the manifold is more directly related to the original physical problem and also easier to compute. The analysis is conducted directly in Hölder spaces, which is often desirable for elliptic equations.

We then use this machinery to construct small bounded solutions to a variety of systems. These include heteroclinic and homoclinic solutions of the anti-plane shear problem from nonlinear elasticity; exact slow moving invasion fronts in a two-dimensional Fisher–KPP equation; and hydrodynamic bores with vorticity in a channel. The last example is particularly interesting in that we find solutions with critical layers and distinctive “half cat’s eye” streamline patterns.

This is joint work with Robin Ming Chen and Miles H. Wheeler.