Global bifurcation for monotone fronts of elliptic PDE

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Abstract

In this talk, we will discuss recent results on global continuation of monotone front-type solutions to elliptic PDEs posed on infinite cylinders. This is done under quite general assumptions, and in particular applies even to fully nonlinear equations as well as quasilinear problems with transmission boundary conditions. Our approach is rooted in the analytic global bifurcation theory of Dancer and Buffoni–Toland, but extending it to unbounded domains requires contending with new potential limiting behavior relating to loss of compactness. We obtain an exhaustive set of alternatives for the global behavior of the solution curve that is sharp, with each possibility having a direct analogue in the bifurcation theory of second-order ODEs.

As a major application of the general theory, we construct global families of internal hydrodynamic bores. These are traveling front solutions of the full two-phase Euler equation in two dimensions. The fluids are confined to a channel that is bounded above and below by rigid walls, with incompressible and irrotational flow in each layer. Small-amplitude fronts for this system have been obtained by several authors. We give the first large-amplitude result in the form of continuous curves of elevation and depression bores. Following the elevation curve to its extreme, we find waves whose interfaces either overturn (develop a vertical tangent) or become exceptionally singular in that the flow in both layers degenerates at a single point on the boundary. For the curve of depression waves, we prove that either the interface overturns or it comes into contact with the upper wall.

This is joint work with Ming Chen and Miles H. Wheeler.