Orbital stability and instability of fractional KdV solitary waves

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Abstract

Consider nonlinear dispersive PDEs of the form

$$\partial_t u = \partial_x (|\partial_x|^\alpha u - u^p),$$

where $u = u(t, x) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is the unknown, $\alpha \in (1/3, 2]$ describes the strength of the dispersion, and $p > 1$ represents a “generic” power law nonlinearity. This family of problems includes a number of important hydrodynamical models, including the famous Korteweg–de Vries ($\alpha = 2, p = 2$) and Benjamin–Ono equations ($\alpha = 1, p = 2$).

It is well-known that (1) has solitary wave solutions, and there is an extensive literature devoted to studying their stability properties. Notably, Bona, Souganidis, and Strauss obtained a general stability/instability criteria that applied for $\alpha \in [1, 2]$. In this talk, we will present a new, more direct proof of this seminal result. Our argument relies on a relaxed version of the Grillakis, Shatah, and Strauss method which can directly treat (1). In fact, we are able to extend the theorem to the fractional regime $\alpha \in (1/3, 1)$. This is joint work with Kristoffer Varholm (NTNU) and Erik Wahlén (Lund).