

Algebra I and Algebra II classes will contain topics from the following list. It will be up to the discretion of the instructor to fashion his syllabus from the topics below or add more topics. Some text books covering these topics are Jacobson, Lang, Hungerford, Dummit and Foote, Lang (Linear Algebra) and Hoffman and Kunze.

Basics of Groups:

Semi-groups, monoids and groups
Homomorphisms and subgroups
Cyclic groups
Cosets
Normal subgroups and quotients
Symmetric, alternating and dihedral groups
Direct products and direct sums

Structure of Groups:

Free abelian and finitely generated abelian groups (The proof of the structure theorem for finitely generated abelian groups may be deferred to the unit on modules.)
Group actions
Sylow theorems
Examples of classification of finite groups of small order
Nilpotent and solvable groups
Jordan-Holder decomposition

Rings:

Rings and ideals
Factorization domains
Localization of rings at multiplicatively closed subsets
Polynomial rings and factorization (eg Eisenstein's criterion)

Fields and Galois Theory:

Field extensions
Fundamental theorem of Galois theory (correspondence between intermediate field extensions and subgroups of the Galois group)
Splitting fields & algebraic closure
Galois groups of polynomials
Finite fields
Theorem on insolubility by radicals of general polynomial equations of degree ≥ 5

Modules:

Module homomorphisms and exact sequences
Free modules and vector spaces
Projective and injective modules
Hom functor and duality
Tensor products and flat modules
Classification of modules over a PID

Linear Algebra:

Linear Transformations
Duals

Determinants

Tensor, symmetric and exterior products

Eigenvalues, Characteristic polynomial and diagonalization,

Bilinear forms, inner product spaces

Linear functionals and adjoints

Hermitian, Unitary and Normal operators

Canonical forms (Jordan form, Rational canonical forms) (using the structure theorem for modules over a PID)

This ends the qualifying exam material. The rest of the class is a brief introduction to Commutative Algebra.

Introduction to Commutative Rings:

Noetherian Rings and Modules

Primary ideals and primary decomposition for Noetherian rings

Integral extensions and the Cohen-Seidenberg (going up/down) theorems

Noether normalization and Nullstellensatz

